Cycle Length Distributions in Graphical Models for Iterative Dec

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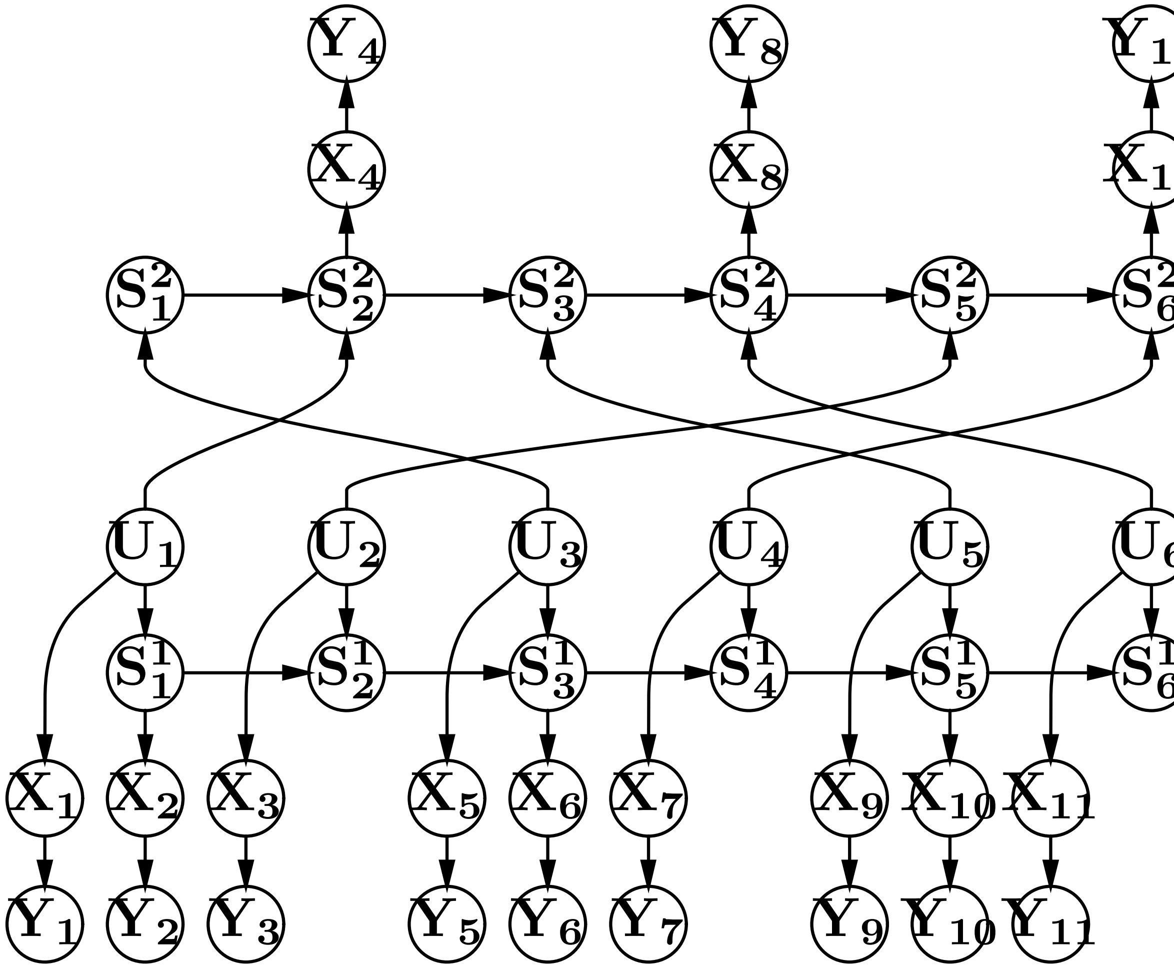
Talk given by *Igor V. Cadez*

Turbo Code: An Error-correcting code Shannon limit

- Near-optimal performance (in terms of bit error rate)
- Theory not well understood
  - Decoding is a special case of local message passing algorithm in directed graphical models, which only proven to work for graphs *without loops*.
  - But the graphical model of Turbo Code has loops.

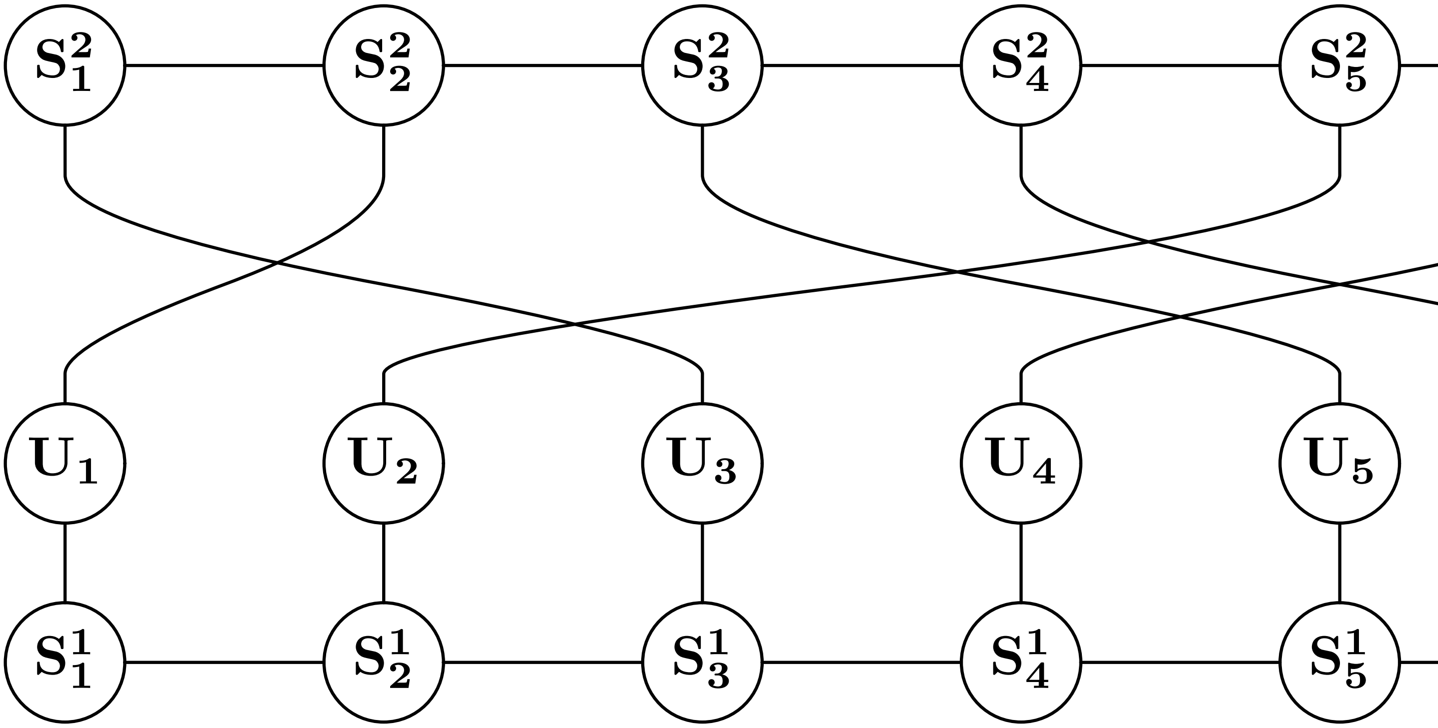
Turbo Code: The Graphical Model



Counting Cycles: Motivation

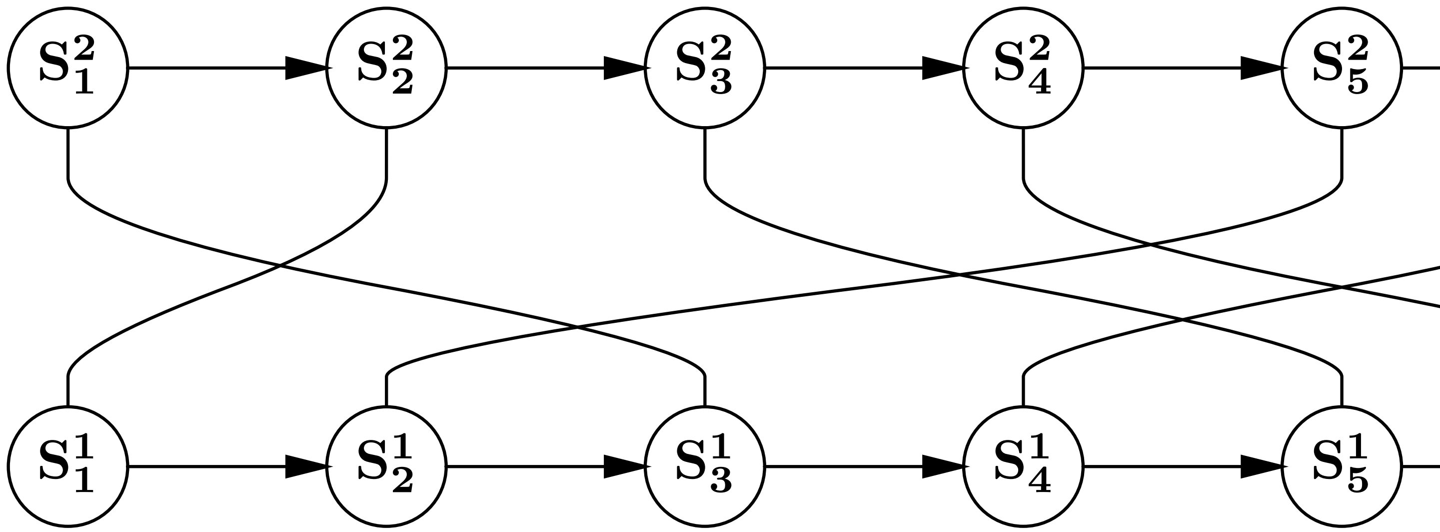
- Loops/cycles in the graphical models introduces "*double-counting*" of evidence.
- Conjecture: doubling-counting effect dies off in long c
- $\Rightarrow$ *How many cycles of length* $\leq k$ *are there at a rand* *chosen node in a typical graph for a Turbo Code?*

The Cycle Structure



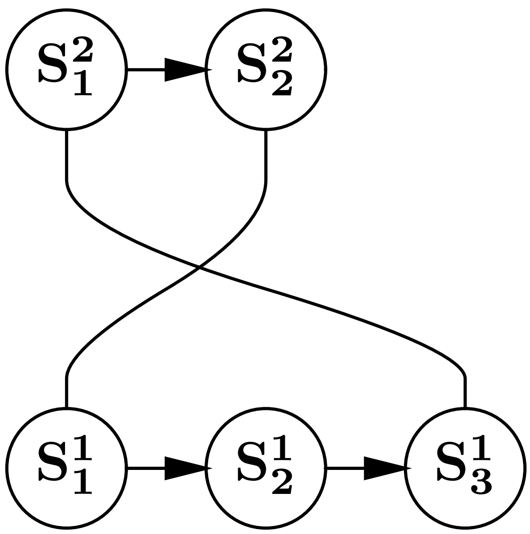
The Simplified Cycle Structure: dropping U

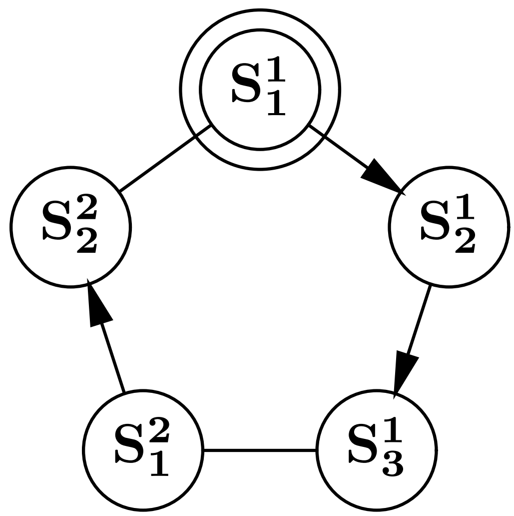
- Edges are labeled $\rightarrow$, $\leftarrow$ (on the chains), $-$ (across th

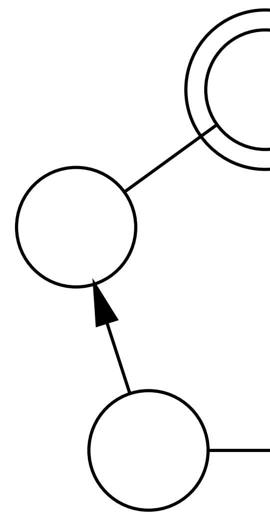


One example cycle: $\mathbf{S_1^1 \rightarrow S_2^1 \rightarrow S_3^1 - S_1^2 \rightarrow S}$

- : Label sequence: $\rightarrow, \rightarrow, -, \rightarrow, -$.







How to count the cycles of length $\leq k$ a
randomly chosen node?

1. Let n be the length of the chain in the graph, i.e. the
   length of Turbo Code.
2. Because the degree of the nodes is 3, there are $\approx 2^{k-2}$
   label sequences of length k.
3. For a given label sequence, the probability of the exis
   the corresponding cycle at the node is $\approx \frac{1}{n}$, for $k \ll n$
4. The probability of no cycle of length k at a node is (1
5. The probability of no cycle of length k *or less* at a no
   $\approx \prod_{i \leq k}(1 - \frac{1}{n})^{2^{i-2}} \approx e^{-\frac{2^{k-1}-4}{n}}$.

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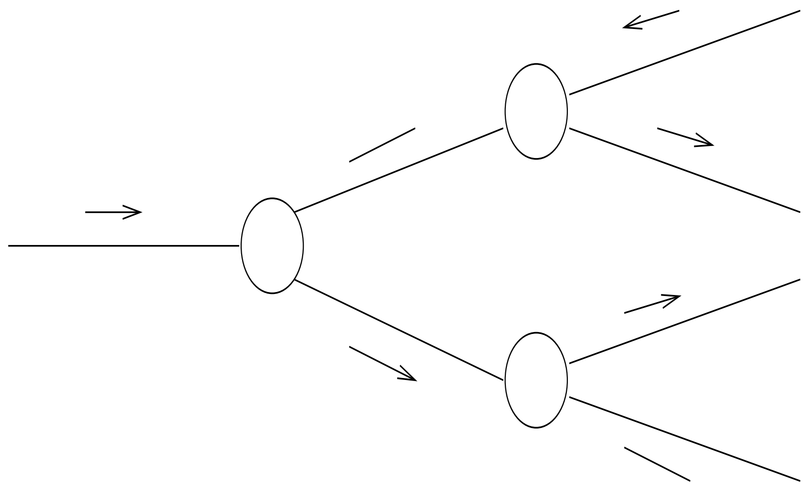
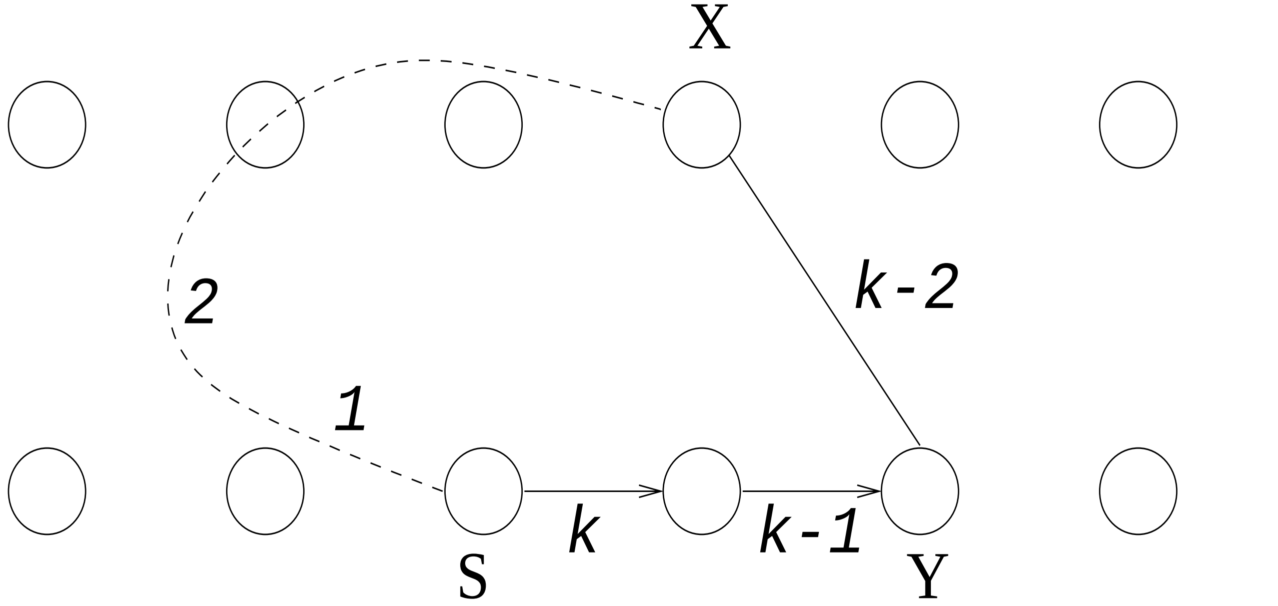


Label Sequence			
$\cdots$	$\rightarrow$	$-$	$\leftarrow$
$\cdots$	$\rightarrow$	$-$	$\rightarrow$
$\cdots$	$\rightarrow$	$\rightarrow$	$\rightarrow$
$\cdots$	$\rightarrow$	$\rightarrow$	$-$

- At a randomly chosen node S, for a given label seque
length k, the probability of the existence of the corres
cycle is $\frac{1}{n}$:
The last across-chains edge XY can go from X to an
the n nodes, instead of Y. So the probability is $\frac{1}{n}$.

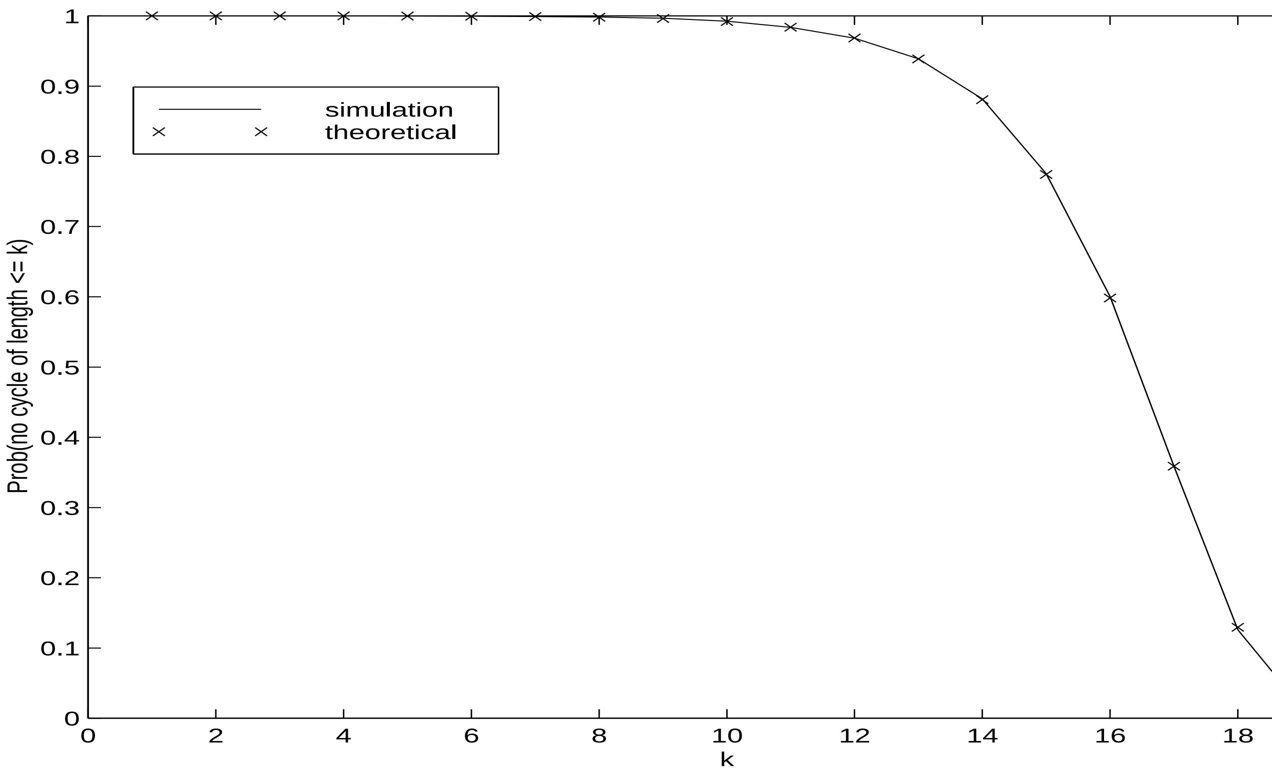


At a randomly chosen node S, with proba
$\approx e^{-\frac{2^{k-1}-4}{n}}$, there is no cycle of length $\leq$

- For a given label sequence of length k, with probabilit the corresponding cycle does not exist at the node S.
- With probability $\approx \left(1 - \frac{1}{n}\right)^{2^{k-2}}$, none of the 2^{k-2} cyc length k exist at the node S. (*Independence assumpti*
- The probability of no cycle of length $\leq k$ at at the no $\approx \prod_{i \leq k}(1 - \frac{1}{n})^{2^{i-2}} \approx e^{-\frac{2^{k-1}-4}{n}}$.(*Independence assump*

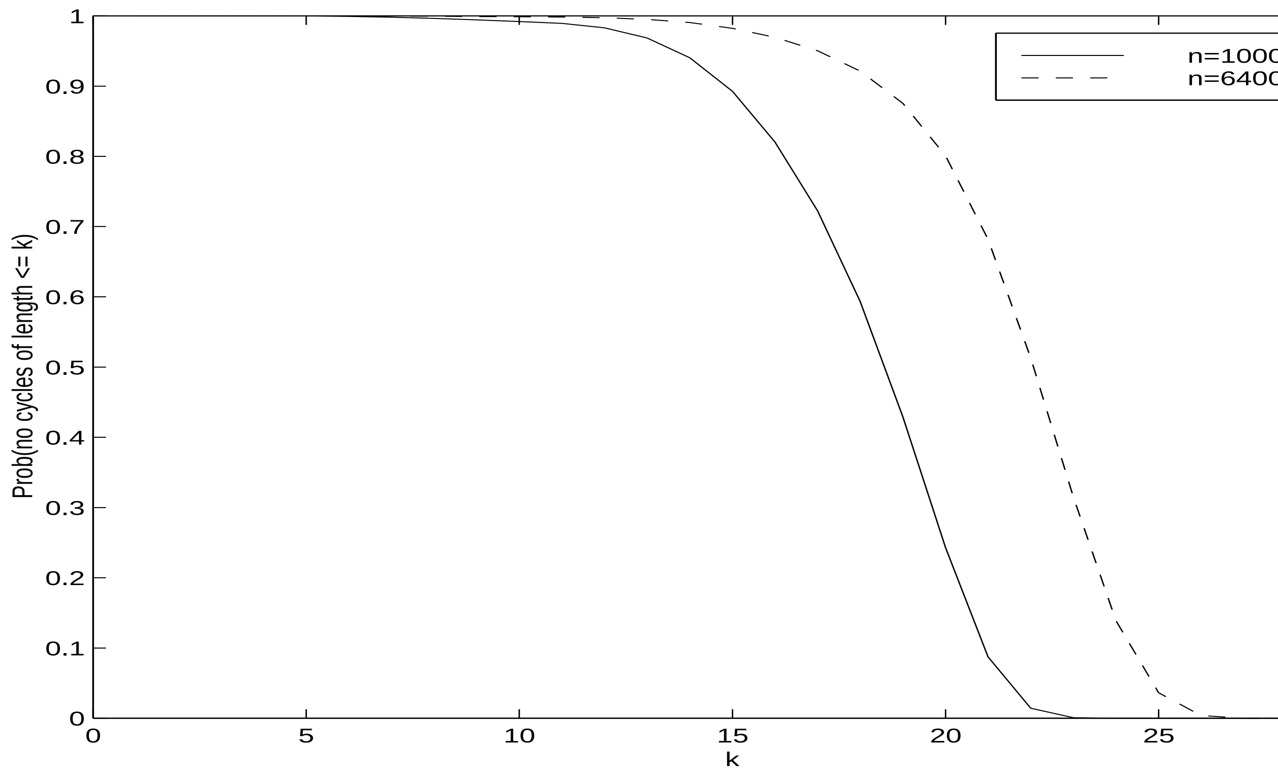
The approximate theoretical results are ve[r]
to simulation results.

Probability of no cycles of length k or less, as a functio[n]



Cycle lengths distribution for the graph models of Turbo Code

Probability of no cycles of length k or less, as a functio



Conclusions and Other Results

- For a randomly chosen node, the probability of a shor (length < 10) is very low (close to 0) and a long cycle close to 1.
- At a randomly chosen node S, for the same k, the pro of no cycle of length $\leq k$ increases with n.
- Let $k_{0.5}$ be such that $p(k_{0.5}, n) = e^{-\frac{2^{k_{0.5}-1}-4}{n}} = 0.5$. T $k_{0.5}$, n need to be increased to n^2: $p(2k_{0.5}, n^2) \approx 0.5$.
- S-random permutation: not much effect on this curve
- Low-Density Parity Check (LDPC) codes: similar resu (similar curve shape), but the independence assumpti accurate (simulation does not agree as well).

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